



K25P 2894

Reg. No. :

Name :

**III Semester M.Sc. Degree (CBCSS – OBE – Reg./Supple./Imp.)
Examination, October 2025
(2023 Admission Onwards)**

**Mathematics/Mathematics (Multivariate Calculus and Mathematical
Analysis, Modelling and Simulation, Financial Risk Management)
MSMAT03C12/MSMAF03C12 : COMPLEX ANALYSIS**

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any 5** questions from the following 6 questions. **Each** question carries 4 marks.

1. Prove that $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi$ if $|z| < 1$.
2. Prove the following : If f is an entire function then f has a power series expansion $f(z) = \sum_0^{\infty} a_n z^n$ with infinite radius of convergence.
3. Find all zeros of the function $f(z) = \sin\left(\frac{1-z}{1+z}\right)$, $|z| < 1$.
4. Let p be a non-constant polynomial and $c > 0$. Show that each component of $\{z : |p(z)| < c\}$ contains a zero of p .
5. State and prove Maximum Modules Theorem (First Version).
6. Prove that $H(G)$ is a complete metric space. **(5×4=20)**

PART – B

Answer **any 3** questions from the following 5 questions. **Each** question carries 7 marks.

7. Prove the following : Let G be an open set and $f : G \rightarrow \mathbb{C}$ an analytic function. If γ is a closed rectifiable curve in G such that $n(\gamma, \omega) = 0$ for all $\omega \in \mathbb{C} - G$ then for $a \in G - \{\gamma\}$, $f^k(a)n(\gamma, a) = \frac{k!}{2\pi i} \int_{\gamma} \frac{f(z)}{(z-a)^{k+1}} dz$.

P.T.O.



8. Prove the following : Let G be a connected open set and let $f : G \rightarrow \mathbb{C}$ be an analytic function. Then the following conditions are equivalent.
- $f \equiv 0$;
 - There is a point a in G such that $f^n(a) = 0$ for each $n \geq 0$;
 - $\{z \in G : f(z) = 0\}$ has a limit point in G .
9. Prove the following : If $\{f_n\}$ is a sequence in $H(G)$ and $f \in C(G, \mathbb{C})$ such that $f_n \rightarrow f$ then f is analytic and $f_n^{(k)} \rightarrow f^{(k)}$ for each integer $k \geq 1$.
10. Prove the following : Suppose $F \subseteq C(G, \Omega)$ is normal if and only if for every compact set $K \subset G$ and $\delta > 0$ there are functions f_1, f_2, \dots, f_n in F such that for f in F there is at least one $k, 1 \leq k \leq n$, with $\sup\{d(f(z), f^k(z)) : z \in K\} < \delta$.
11. State and prove the argument principle. (3×7=21)

PART – C

Answer **any 3** questions from the following 5 questions. **Each** question carries **13** marks.

12. State and prove the third version of Cauchy's theorem.
13. a) State and prove the Residue theorem.
- b) Prove that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$.
14. a) State and prove the Rouché's theorem.
- b) State the fundamental theorem of Algebra. Prove the same by using Rouché's theorem.
15. State and prove the Riemann Mapping theorem.
16. a) State and prove Schwarz's lemma.
- b) State and prove Montel's theorem. (3×13=39)
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